Examiners' Report

Principal Examiner Feedback

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Pearson Edexcel International A Level

In Pure Mathematics (WMA12/01)

Paper: WMA12/01

General

The paper was a little bit more challenging than some recent series with a few places where candidates had to do more than just apply a routine procedure. Questions with unknowns that needed to be found or shown, as opposed to simply carrying out a process, occurred in questions 1, 2, 6 and 9, for example. Use of logs rules within integrals, proof and trigonometry also caused difficulties for many students with a model score of 0 marks on questions 3, 9 and 10 respectively. That question 3 proved so challenging was surprising, whereas for question 10, being the last question and candidates running out of time must have been a contributing factor, as this question did not have any unexpected twists.

The continued effects of Covid on students learning was apparent and should be taken into consideration when reading through this report.

Report on individual questions

Question 1

For the first question, which was a relatively standard twist on a binomial question, there were a surprising number making no or little attempt and unable to access the question, with nearly 20% scoring no marks. Part (b) provided an early challenge but the first 4 marks were highly accessible, and indeed, when once underway 80% of candidates scored at least 3 marks, with 30% scoring the modal 6/6 marks.

(a) Most who attempted the question were able to set up or imply at least one correct equation, usually $16k = -4$ to obtain $k = -1/4$, though a small number of candidates were incorrect in this equation by did set up the correct equation in p and k for the x^2 term. There were more errors finding p with some having problems with the binomial coefficient or squaring k.

(b) Most were able to achieve the first M mark, usually for finding the value of $2p$, though again a small number of cases focussed on the less obvious term but missed the obvious one. However, fewer than 50% made further progress, making no attempt to find the other term in x^2. That the x^3 term had not been asked for meant many missed it was needed, though it is hoped that candidate would realise they need multiple terms from expanding brackets to get all the coefficients of a certain term. Of those who realised what they needed to do, some made errors in finding the coefficient so losing the A mark. Most who got values for both x^2 terms added the two together though a few lost the third mark as they did not add or added wrongly or missed the negative sign. A lack of understanding of the difference between "term" and "coefficient" lost some the final A mark for only finding the coefficient of x^2 and not writing down the whole term.

Question 2

This was the best performing question on the paper with a mean score of about 3.5/5, and over 45% achieved the modal full mark score. The unknown constant caused problems for a few students, with powers sometime lost in evaluating the second term, but most showed the correct method. Nevertheless, on this question just over 20% of candidates failed to score any marks, while 2, 4 or 5 were the other common totals.

(a) Most candidates tackled this part well, managing to give $u_3 = 6k^2 + 3k + 3$ or the equivalent, with 77% scoring the first M and just under 75% scoring the A. Neglecting to square the k and achieving $9k + 3$ was often the reason for M1A0, though isw was applied in many such instances where a correct expression was seen first. Another common error was to omit the second occurrence of "+3", resulting in $6k^2 + 3k$, and there were some cases where $u_2 = 6k + 3$ became $9k$ so $u_3 = 9k + 3$ became $12k$, showing a weak understanding of basic algebra.

The candidates who did not achieve any marks offered a variety of different incorrect methods such as writing $u_3 = ku_2 + 3$ with no further substitution, or assuming the rule was only to add 3 for each application. In some instances, u_3 was not seen until part (b), which earned no marks in (a) as candidates needed to demonstrate an understanding of what 'u_3 in terms of k' meant.

(b) This part was less successfully completed than part (a), with about 65% scoring the two method marks, and less than 50% achieving all three. The most common errors were either setting u_3 equal to 117 rather than the sum, or attempting to use the formula for the sum to n terms of an arithmetic series. Those who did not have a quadratic expression for (a) were only able to access the first mark of (b).
However, many candidates did manage to put the sum of their first three terms equal to 117 and, despite some slips in algebra or with signs, managed to get a 3 term quadratic which they endeavoured to solve. The most frequent arithmetic slip was omitting a 3 from simplifying resulting in a $6k^2 + 9k - 108 = 0$. The method for solving a 3 term quadratic is well known, so most who achieved one having scored the first M also scored the second, though a small number of candidates applied an incorrect method for solving. It was not uncommon to see just the solutions from a calculator approach.

The final mark was scored by most of those who had achieved a correct quadratic to solve, but a failure to omit the negative solution did cause many to lose the last A mark. Candidates need to show an awareness of constraints on constants given in the question.

Question 3

This proved a very challenging question for candidates with only questions 9 and 10 seeing a mean score with lower percentage of the maximum. Candidates often score well on questions on the trapezium rule and indeed that is where most of the marks were gained, with the B and M marks in part (a) being the only marks with over 50% success rate in this question. The application to the integrals in part (c) proved very challenging with less than 40% scoring the mark for (i) and fewer than 25% making significant progress with part (ii). The modal mark was 0/10, scored by nearly 30% of candidates.

(a) Fewer than 60% of candidates were able to apply the trapezium rule correctly with the specified number of strips, with success rate of around 55% for each of the first two marks. Identifying the correct value of h for the number of strips required was the first hurdle, with many using $h = 4$, perhaps mistaking 'four strips of equal width' as 'strips of width four'. The M mark was still accessible even with an incorrect width, yet still proved more difficult than envisaged. Perhaps candidates would have made more of a success of this had they been given a table of values, but they should be able to apply the trapezium rule to a given problem without one. Some wrote the rule out using log terms but the majority used decimal values.

Only 40% were able to carry out the rule fully correctly to achieve the A mark. For those scoring the first two marks but failing to gain the A, this was usually due to calculation slips, particularly for the final term with 1.46 seen instead of 1.146. The problem for others was that they could not successfully work out that 5 values of $y = \log_{10}x$ were required at $x = 2,3,8,11$ and 14 and so had incorrect values in their expression.

A small amount of candidates attempted to integrate instead of use the trapezium rule, but this was uncommon.

(b) About 50% of candidates gave a correct answer for this part, expressed in many different ways but essentially with the correct meaning. Some candidates, though, did not understand the question and instead explained how the trapezium rule is used to estimate areas under curves or suggested giving intermediate values to more decimal places or analytically integrating instead.

(c) This proved to be a lot more challenging than expected, with many not realising what to do at all and offering no attempt, some repeating the trapezium rule rather than using the answer to part (a), while many used a calculator to find the actual value of the area. There were also incorrect attempts at integrating, such as $\log_{10}\sqrt{x} \rightarrow \frac{2}{3}\log_{10} x^{3/2}$

Although the mark was awarded in (i) if the trapezium rule was repeated, such was not acceptable for (ii), and so fewer had access to this part. Many candidates simply omitted this part, while again attempts at using the calculator to obtain a more accurate value, attempting to integrate the function or redoing the trapezium rule were commonly seen. Even where a recognition of the need to apply the log laws was seen, the attempts were not always successful, with $3\log_{10} 100x$ fairly common, from which it was possible to achieve the method mark the sum law was also applied and an attempt at integrating the constant term and using the answer of (a) was used. Of the candidates who were able to apply the logarithm rules correctly some did not integrate the 2 and so lost the method mark.

Question 4

In contrast to question 3, this question provided good access for candidates, a familiar topic, with over 75% of candidates scoring 4 or more marks, and only a very small number of candidates scoring no marks at all. The last two marks did have some demand to them, with just under 50% accessing the final M, and about half of these going on to achieve the A mark.

(a) Although about 80% were able to get this mark many did not realise the answer could be written down directly: the question did say "state" to indicate that working was not needed. Many correct responses involved unnecessary expanding, simplifying and dividing using long division or substituting 3/2 to get the remainder of –21. The latter methods sometimes led to slips.

(b) Despite the question's clear instruction to use the factor theorem, numerous candidates nevertheless tried to use long division and hence gained zero marks. However 80% did attempt the factor theorem, but only a little over half of these went on to secure the accuracy mark, mainly due to lacking a conclusion that '$(x - 3)$ is a factor' M1A0 was a common mark for this part as a result.

(c) (i) Even when no other marks were gained, the B mark was often awarded when the candidate expanded f(x) to give the correct cubic, with over 90% securing this mark. Both methods, of factorisation and long division, were seen frequently. One advantage of inspection was that they did not lose the final mark by failing to write the linear and quadratic factors together on one line, which many who attempt long division failed to do. Some candidates had correct workings in part (b) having used long division there but could only access mark if it was referred to in part (c), which was not always the case. There were also some candidates who jumped to a fully factorised (with real and complex roots) presumably via use of a calculator, which should be discouraged especially when questions specify a "hence" to demand a particular method.

(ii) The most common method attempted here was finding the discriminant, although many did not seem clear on why this was significant. Many also attempted to apply the quadratic formula, although some stopped when they could not take the square root without completing. The majority applied $b^2 - 4ac$ in some form and generally reached –31. If considering the discriminant only they usually stated this was less than 0. Although 50% achieved the M mark, to achieve the A mark was quite demanding and often one of the required elements for it was missing with fewer than 30% attaining the final mark. There were, however, some very good responses to this part with well-reasoned and precise solutions.

Question 5

Another very well approached question with 5 or 6 marks (out of 6) being attained by over 60% of candidates and less than 10% scoring 0 or 1. Generally, candidates knew the formulae for *n*th term and S_n for Arithmetic Sequences and were able to apply them. The demands of a 'show that' in (b), and the need to select which root in (c) provided the challenge in this question.

(a) Most used the method in the scheme proceeding via $a+(n-1)d$ with their values. They generally had two of the three *a*, *n*, *d* correct over 90% achieved the first M mark and a little over 80% achieving both marks. A fairly common mistake was to use $d = 15$ instead of $d = -15$, with errors in the other two variables being rare. Some of these realised their error when they could not achieve the required equation in (b) and went back to find their mistake and correct, though some realised their error but tried to "fudge" the answer rather than identifying how to correct it.

Those who attempted by repeated subtraction usually got the correct answer though some stopped after year 13 or year 15. Only a very small number of candidates thought the sequence was geometric and used completely wrong formulae, thus losing almost all the marks for the question.

(b) Over 60% of candidates gained full marks for this part, with a correct formula and clear steps showing how they got to the required equation. Marks were lost for careless manipulation of terms, missing "=7770" or missing "=0" on the final line. Miscopying was another cause of lost marks and incorrect bracketing also led to errors. It was pleasing to see that most candidates knew that they had to show at least one step between the starting point and the given result, with only a few jumping from the formula with the values substituted to the required equation. Many showed more steps and some even explained what they were doing at some of the steps.

(c) Most knew to solve the quadratic in (b) and found the correct values, often by calculator, though some did show factorisation, but less than 50% then selected the correct value of two, as most either did not reject 37 at all, leaving both 28 and 37, or chose 37 as their answer. Some candidates not alone selected the correct root, but also correctly explained why the answer was 28, which was pleasing to see.

Question 6

Circles work is often a difficult question for candidates, and this proved no exception, with both parts causing difficulties. The model mark was comfortably 2 out 8 marks, scored by 25% of candidates, though the mean was 3.8. The first 2 marks were the ones most often scored.

(i) About 80% achieved both marks in part (a) but less than 30% made any progress with (b). Although there were some unconvincing attempts at completing the square candidates could generally identify the centre and the first two marks could be scored by sight of the correct coordinates. Sometimes that was all that was seen. Those that did not score the first two marks either did not complete the square correctly on both x and y, usually yielding (10, 6) or simply had no attempt at all. Sign errors in the coordinates were rare but not unseen.

For part (b) most did not realise that an inequality was needed, trying instead to identify a value for k, so could not access either mark. A careful reading of the question to know what type of answer to expect is advised. There was little recognition that the radius needed to be positive and that $k + 61$ represented r^2. Some who included an inequality tried $25 < k < 36$, while some had $k > 61$.

(ii) Less than 50% of candidates made successful progress in this part, though when they did usually the first three marks, or at least the two Ms, were gained. It is also notable that a minority of candidates did not realise that the two parts were independent, and tried to apply their centre, and sometimes a radius, from part (i) in part (ii). There is a need to cultivate an awareness of the structure of a question.

The most favoured method relied on them finding both the radius and centre of the circle C_2, then going on to use both correctly to find p. It was quite common to see them attempt to use the diameter as their radius, from which only the second M could be gained. Students who found the correct centre and radius usually then used the point $(p, 0)$ to form a quadratic, but often numerical errors in rearranging stopped them gaining full marks.

Candidates should be reminded to quote formulas used as there were examples of incorrect substitutions which lost the method mark without evidence of a correct formula, for example $\left(\frac{8+2}{2}, \frac{10-14}{2}\right)$ or $\sqrt{(-2-8)^2+(10-14)^2}$.

Other slips included setting "y – 2" = 0 where the second term in the circle equation became became 0 rather than 4, and a few did not reject the negative answer.

There were very attempts at the alternative methods which made use of the angle in a semi-circle or Pythagoras, and mixed success when these were used. A few attempted to find p by using the gradient method but incorrectly multiplied the gradients of two sides which were not perpendicular, typically QR and PX.

Question 7

This question was quite challenging with some candidates not attempting it at all and many others scoring only the B mark for (ii)(b), using the calculator to find the roots. The modal mark of 1 was scored by 14%, with 12% scoring 0 marks. Part (ii) was more accessible than part (i) overall.

(i) Most candidates who attempted this part were able to score the first mark, with 65% success rate. However, most were unable to progress successfully past this point with only 20% achieving the method, and 15% the correct answer.

There were many errors in index or log work, with progression from $4\times 6^{n-1}$ to 24^{n-1} being very common. Incorrect application of the power law after taking logarithms was also common, and $\frac{10^{100}}{4}=25^{100}$ was also seen numerous times. Other candidates were unable to deal with the power of 100 by using the logarithm rule for powers at all and stopped short of finding n. Some gave up once their calculator displayed 'math error' thinking it could not be solved, although there was evidence some may possess calculators which can deal with numbers as large as 10^{100}.

A few successful candidates simplified $10^{100}/4$ by writing it as 2.5×10^{99} which their calculators could deal with, or by splitting the power into smaller powers of 10 and correctly using logarithms to calculate these. There were also a few who incorrectly tried to use the formula for a sum of a geometric series, rather than an nth term formula.

(ii) This part provided better access to marks with the main point of difficulty being in selecting and justifying the correct value for r. Of those candidates who attempted (a), many successfully reached the required answer, though some candidates lost the final mark for slips in their working such as a sign error or missing a r in one of the relevant terms. Over 60% score the first two marks, with less than 55% securing the third. A small number tried to use the sum of n terms formula rather than the sum to infinity. Some candidates formed an equation in terms of a instead of r and did not proceed to change variable.

Part (b) was the most successful of this question with 80% securing the mark. Some showed all their working even though it was only a one mark question so the answers could have been stated.

Less than 25% managed to answer part (c) correctly. Many assumed that r had to be positive for there to be a sum to infinity and chose $\frac{6}{5}$. Those who selected the correct value often could not correctly explain why with r needing to be less than 1 given as a common reason, not $|r| < 1$. Some selected $r=-\frac{1}{5}$ because the geometric series was convergent which is not considered a complete explanation so did not gain the mark, though solutions which reasoned out that all terms would be negative with a positive ratio and negative second term were acceptable.

Many who attempted (d) were able to get at least the method mark for a correct substitution and if using the correct value for r often went on to get the correct value. Only about half of those who secured the method had a correct value, though. There were also candidates who did not state the formula and made mistakes substituting in the values especially if using $r = -\frac{1}{5}$ where they sometimes calculated $\frac{a(1+(-1/5)^4)}{1+1/5}$ so gained no marks for this part. The advice is to write down a correct formula before attempting to use it as then the method can be awarded if slips are made.

Question 8

Whilst there were a lot of blank responses, the overall success of this question was generally good, with a mean score of 5.36 out of 10. The modal score was full marks, achieved by 18%, but the next most common score, at just under 18%, was 0 marks, with an even distribution of marks between, though slightly favouring the higher marks.

(a) Successfully completed by over 60% of candidates, though as usual with 'show' questions some candidates tried to make their working fit to the given value of 28 following errors, rather than checking the work carefully. Over 70% started with a correct attempt at differentiation, with most evidencing an attempt to substitute $x = 2$ and setting equal to zero. The setting equal to 0 was sometimes only applied by working, forfeiting the accuracy mark.

Incorrect attempts usually involved using $x = 2$ in the equation for y instead of differentiating first (and still somehow managing to rearrange to give $k = 28$).

(b) Less than 50% of candidates realised what was needed in this part and attempted to factorise the deritaive. It was common to see an attempt at the second derivative instead of factorizing $\frac{dy}{dx}$ to find the critical values. Some erroneously thought the lower region was $0 < x < 2$ and gave only this as the solution with no attempt at the upper end.

For those who did achieve the correct critical values, the A mark was not always earned because candidates were not always sure how to find the actual range of x values, with the "$0 < x$" being common to add, while some took the central interval. When the correct ranges of values were achieved, most used inequalities, though a few using other correct set notation.

(c) Most candidates were able to integrate correctly with 60% gaining the first 2 marks. Some then simply substituted limits of 2 and 0 to give an answer of 32 and stopped there while some correctly found the y value when $x = 2$ but then either failed to use the value at all in their working, or occasionally, used 68/3 as one of the limits for the definite integration. Those who found the area of the rectangle and subtracted the integral were less likely to make slips than those who used a line – curve approach as sign errors were then often made.

Over 50% correctly found the 68/3, but less than 45% used a correct method, with 33% reaching the final answer successfully. Incorrect attempts at the method sometimes involved lengthy work to try and find the other intersection point of l and C and use this as a limit, or attempting the area a triangle rather than rectangle.

There were only a minority candidates who gave a "correct" answer purely from use of a calculator with no algebra to support it.

Question 9

This question was expected to prove challenging to candidates, with proof being one of the new topics on the current specification. There were a considerable number who did not attempt this question as well as many who attempted but scored no marks, with 47% scoring the mode of 0 out of 4, and less than 25% scoring more than the mean score of 1 mark. Overall, this was the worst performing question on the paper.

(a) There has been some precedent for this type of question past series, and though only a third of candidates were able to score the first mark, there were many who attempted an algebraic proof. Squaring to only a two term expression was common, though, losing the method mark. A general lack of rigour in presentation and an unfamiliarity with the requirements of a robust proof were evident across most scripts seen. Those who knew how to present a proof were generally more successful.

Though many candidates did attempt some kind of algebraic proof, there was also a large number who considered that several numerical examples of the inequality holding constituted a proof.

The most common successful approach was to attempt to square both sides of the given inequality and arrive at a true statement (method Alt 1 in the scheme). However, it was rare for such approaches to achieve all three marks as full conclusions were seldom given, and students do not realise the flow of direction of implications matters.

(b) Since any two negative numbers provide a counter-example to the statement, the mark required a correct justification for the example given, and little over a third of candidates achieved this. A clear statement showing the substitution followed by a simple 'not true' would have sufficed, but even such a conclusion was often lacking, and errors were made in the substitution in some cases.

Despite the directive to prove by counter example, there were also attempts at algebraic proofs made by several candidates. Also, a frequently seen explanation was that $\sqrt{xy}$ would result in a 'math error' or 'wasn't possible because you cannot take a square root of a negative number' (even though a product of two negative numbers will be positive). The reliance on a calculator and misunderstanding of basic algebra is something was disappointing to see.

Question 10

This question was surprisingly poorly performing, though with many blank responses this may indicate that some students were running out of time to complete. Fully correct solutions to both parts were very rare and the modal mark was again 0 marks, scored by 35% of candidates, and 70% scored fewer than 5 marks. The trigonometric methods in each part were not unusual, but basic algebra let down many.

(i) Even when attempts were made, many errors were seen with less than 60% of candidates scoring the first M. Due to the dependent nature of the marks in the question, this meant no marks would be scored in this part. Common errors included: calculating $\tan^{-1}(3)$ and completely ignoring the square, or then going on to square root the result; splitting the angle up into $\tan^2(2x) + \tan^2(\pi/4)$; subtracting π /4 and dividing by 2 before taking arctan; attempts to replace tan by sin/cos which either used an incorrect identity, or went wrong in algebraic manipulations to never reach and angle for $2x + \pi/4$.
Of those who secured the first method, most went on to score the next M and A mark, though some did not go on to find a value for x or used an incorrect order of operation. There was less success in closing out the part as many ignored the negative root thus forfeiting the final two A marks. Even when the order of operations was correct and $\pm\sqrt{3}$ was considered, many failed to find all four roots and thus lost the last A mark with the $-\frac{11\pi}{24}$ being the most common angle to omit.

Successful attempts replacing $\tan\left(2x+\frac{\pi}{4}\right)$ by $\frac{\sin\left(2x+\frac{\pi}{4}\right)}{\cos\left(2x+\frac{\pi}{4}\right)}$ were seen, but because it was necessary to reach the stage of taking arcsin or arccos in order to gain the first M mark, most attempts at this did not succeed.
Only positive to note was that only a very small number of responses were seen using degrees. These generally scored 1 or 0, though it was not unknown for a candidate to work entirely in degrees before converting back to radians.

(ii) This part was less well attempted than part (i) with only just over 40% scoring the first method mark to access the question. However, a marginally higher percentage of candidates (just over 10% compared to just under) managed to achieve the correct angles, though not all remembered the 180°.

To achieve the first M, the brackets needed to be expanded to at least three terms and the Pythagorean identity correctly applied. The commonest failing here was once again squaring a bracket resulting in only 2 terms. Of the students who expanded the square bracket correctly, many either did not attempt to apply the Pythagorean identity or used an incorrect identity.

Where the correct first stage was achieved, only about half of these candidates scored the next M. The main reason for this was due to replacing the term in $\sin^2\theta$ rather than $\cos^2\theta$ and stopping at this point when they could not see how to proceed. Since solving such quadratic equations in sin and cos is common in exams, this again may be best put down to time pressures at the end of an exam.

When a suitable equation was reached, there was a tendency to divide by $\sin\theta$ rather than take it out as a factor so even when the candidate correctly achieved 53.1° and 233.1° few included 180° and so lost the B mark, though as many students spotted the 180° but failed to find the other values.

Only a very small minority attempted the alternative version using $2\sin\theta - \cos\theta = \pm 1$, and when this was attempted correctly they usually had extra incorrect solutions in the range. There were also very few attempts at the $R\sin(\theta - \alpha)$ approach seen.